https://doi.org/10.31995/voyager.2024.v15.002

Marginal Convective Instability of MHD Micropolar Fluid Layer Heated From Below

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Reference to this paper should be made as follows:
Received: 20.10.2024
Approved: 20.11.2024

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Voyager: Vol. XV, 2024
Article No. 02 pp. 009-020

Online available at:
https://anubooks.com/journal-volume/voyager-vol-xv-2024

DOI: *https://doi.org/10.31995 voyager.2024.v15.002*

Abstract

A theoretically investigation is worked out in micropolar fluid layer heated from below in the presence of uniform horizontal magnetic field. The governing differential equations are converted into non-dimensional form using suitable non-dimensional variables and then solved by normal mode technique and obtained thermal Reyleigh number. On the basis of this Rayleigh number stability criteria is examined and predict the behaviour of influenced parameters like magnetic field parameter, coupling parameter, micropolar heat conduction parameter and micropolar coefficient are shown through graphs.

Keywords:

Marginal Instability, MHD, Micropolar fluid, Stationary Convection.

1. Introduction:

The convective instability of a fluid layer heated from below has been studied by many researchers. **Bénard [1]** examined thermal instability using experiment of a fluid layer heated from below. The theoretical aspect of Bénard's experiment has been studied by **Rayleigh [2]** and received a considerable importance due to its relevance in various fields such as chemical and industrial engineering, soil mechanics, geophysics etc. The main objectives of the various studies related to the convective instability, in particular, is to determine the critical Rayleigh number at which the onset of instability sets in either as stationary convection or through oscillations. The Rayleigh-Bénard convection in micropolar fluids heated from below has been extensively studied by **Ahmadi[4]**, **Datta** and **Sastry[3]**, **Bhattacharyya** and **Jena[7], L.E. Payne** and **B. Straughan[5]**. The common results of all these studies are found that the stationary convection is the preferred mode of instability and the microrotation has a stability effect on the onset of Rayleigh-Bénard convection. An excellent review as well as large number of new developments are given by **Chandrasekhar [6]** in his book on hydrodynamic and hydromagnetic stability. In these methods of stability study a linear theory is usually employed *i.e.*, the equations governing the disturbances are linearized and then the grow or decay of the disturbances is studied. The effect of a magnetic field on the onset of convection in a horizontal micropolar fluid layer heated from below has also been investigated by several researchers. The extension of micropolar flows to include magneto-hydrodynamics effects is of interest in regard to various engineering applications such as in the design of the cooling systems for nuclear reactors, MHD electrical power generation, shock tubes, pump, flow meters etc. The effects of throughflow and magnetic field on the onset of Bénard convection in a horizontal layer of micropolar fluid confined between two rigid, isothermal and microrotation free, boundaries have been studied by **Narasimha Murty [9]. Z Alloui** and **P. Vasseur [10]** studied onset of Rayleigh-Bénard MHD convection in a micropolar fluid**. R.C. Sharma** and **P. Kumar[8]** studied the effect of magnetic field on micropolar fluids heated from below and they also studied the effects of magnetic field on micropolar fluids heated from below in porous medium. In both papers, they found that in the presence of various coupling parameters, magnetic field has a stabilizing effect on stationary convection. In this article I studied convective instability of micropolar fluid layer in the presence of horizontal magnetic field and it is found that the Chandrasekhar number has a key role in the investigation of the nature of magnetic field.

2. Mathematical Formulation:

Consider an infinite, horizontal, electrically non-conducting, incompressible micropolar fluid layer of thickness *d*. This layer is heated from below such that the lower

https://doi.org/10.31995/voyager.2024.v15.002

boundary is held at constant temperature $T = T_0$ and the upper boundary is held at fixed temperature $T = T_1$ therefore, a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained. The physical configuration is one of infinite extent in *x* and *y* directions bounded by the planes $z = 0$ and $z = d$. The whole system is acted on by gravity force $\bar{\mathbf{g}}(0, 0, -g)$.

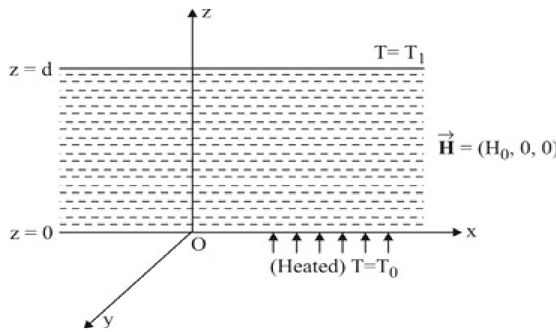

Fig. 1

A uniform magnetic field $\vec{\mathbf{H}} = (H_0, 0, 0)$ is applied along *x*-direction. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The governing equations, describing the system behaviour following Boussinesq approximation, are as follows:

$$\nabla.\vec{\mathbf{q}} = 0 \tag{1}$$

$$\rho_0\left[\frac{\partial \vec{\mathbf{q}}}{\partial t} + (\vec{\mathbf{q}}.\nabla)\vec{\mathbf{q}}\right] = -\nabla p - \rho\, g\, \hat{e}_z + (\mu + \zeta)\nabla^2\vec{\mathbf{q}} + \zeta\nabla\times\vec{\mathbf{N}} + \frac{\mu_e}{4\pi}(\nabla\times\vec{\mathbf{H}})\times\vec{\mathbf{H}} \tag{2}$$

$$\rho_0\, j\left[\frac{\partial \vec{\mathbf{N}}}{\partial t} + (\vec{\mathbf{q}}.\nabla)\vec{\mathbf{N}}\right] = (\alpha' + \beta')\,\nabla(\nabla.\vec{\mathbf{N}}) + \gamma'\nabla^2\vec{\mathbf{N}} + \zeta(\nabla\times\vec{\mathbf{q}} - 2\vec{\mathbf{N}}) \tag{3}$$

$$\rho_0 C_v\left[\frac{\partial T}{\partial t} + (\vec{\mathbf{q}}.\nabla)T\right] = \chi_T\,\nabla^2 T + \delta(\nabla\times\vec{\mathbf{N}}).\nabla T \tag{4}$$

$$\rho = \rho_0[1 - \alpha(T - T_0)] \tag{5}$$

$$\frac{\partial \vec{\mathbf{H}}}{\partial t} = \nabla\times(\vec{\mathbf{q}}\times\vec{\mathbf{H}}) + \gamma_m\,\nabla^2\vec{\mathbf{H}} \tag{6}$$

$$\nabla.\vec{\mathbf{H}} = 0 \tag{7}$$

Where $\vec{\mathbf{q}}$, $\vec{\mathbf{N}}$, p, ρ, ρ_0, μ, ζ, μ_e, j, α', β', γ', C_v, T, t, χ_T, δ, α, T_0, γ_m and $\hat{\mathbf{e}}_z$ denote respectively fluid velocity, microrotation, pressure, fluid density, reference density, fluid viscosity, coupling viscosity coefficient, magnetic permeability, microinertia coefficient, micropolar viscosity coefficients, specific heat at constant volume, temperature, time, thermal conductivity, micropolar heat conduction coefficient, coefficient of thermal expansion, reference temperature, magnetic viscosity and unit vector along z-direction.

3. Basic State of the Problem:

The basic state of the problem is assumed to be

$\vec{\mathbf{q}} = \vec{\mathbf{q}}_b = (0, 0, 0)$, $\vec{\mathbf{N}} = \vec{\mathbf{N}}_b = (0, 0, 0)$, $p = p_b(z)$, $\rho = \rho_b(z)$ and
$\vec{\mathbf{H}} = \vec{\mathbf{H}}_b = (H_0, 0, 0)$

Using this basic state, equations (1) to (7) reduce to

$$\frac{dp_b}{dz} + \rho_b g = 0 \tag{8}$$

$$T = -\beta z + T_0 \tag{9}$$

$$\rho = \rho_0(1 + \alpha\beta z) \tag{10}$$

4. Perturbation Equations:

Let $\vec{\mathbf{q}}' = (u', v', w')$, $\vec{\mathbf{N}}' = (N'_x, N'_y, N'_z)$, p', ρ', θ and $\vec{\mathbf{h}} = (h_x, h_y, h_z)$ represent the perturbation in $\vec{\mathbf{q}}'$, $\vec{\mathbf{N}}$, p, ρ, T and $\vec{\mathbf{H}}$ respectively, then the new variables become
$\vec{\mathbf{q}} = \vec{\mathbf{q}}_b + \vec{\mathbf{q}}'$, $\vec{\mathbf{N}} = \vec{\mathbf{N}}_b + \vec{\mathbf{N}}'$, $\rho = \rho_b + \rho'$, $T = T_b + \theta$, $\vec{\mathbf{H}} = \vec{\mathbf{H}}_b + \vec{\mathbf{h}}$.

Making use of these new variables and equations (8) to (9), equations (1) to (7) in linear forms become

$$\rho_0 \frac{\partial \vec{\mathbf{q}}'}{\partial t} = -\nabla p' + \rho_0 g\,\alpha\,\theta\,\hat{\mathbf{e}}_z + (\mu + \zeta)\nabla^2 \vec{\mathbf{q}}' + \zeta \nabla \times \vec{\mathbf{N}}' + \frac{\mu_e}{4\pi}(\nabla \times \vec{\mathbf{h}}) \times \vec{\mathbf{H}}_b \tag{11}$$

$$\rho_0\, j \frac{\partial \vec{\mathbf{N}}'}{\partial t} = (\alpha' + \beta')\nabla(\nabla.\vec{\mathbf{N}}') + \gamma' \nabla^2 \vec{\mathbf{N}}' + \zeta(\nabla \times \vec{\mathbf{q}}' - 2\vec{\mathbf{N}}') \tag{12}$$

$$\rho_o C_v \left[\frac{\partial \theta}{\partial t} + (\vec{\mathbf{q}}'.\nabla) T_b \right] = \chi_T \nabla^2 \theta + \delta(\nabla \times \vec{\mathbf{N}}').\nabla T_b \tag{13}$$

$$\frac{\partial \vec{\mathbf{h}}}{\partial t} = (\vec{\mathbf{H}}_b.\nabla)\vec{\mathbf{q}}' + \gamma_m \nabla^2 \vec{\mathbf{h}} \tag{14}$$

$$\nabla.\vec{\mathbf{h}} = 0 \tag{15}$$

Using the following non-dimensional variables

$x = dx^*,\ y = dy^*,\ z = dz^*,\ \vec{\mathbf{h}} = H_0\vec{\mathbf{h}}^*,\ \vec{\mathbf{q}}' = \frac{K_T}{d}\vec{\mathbf{q}}^*,\ t = \frac{\rho_o d^2}{\mu}t^*,\ p' = \frac{\mu K_T}{d^2}p^*$

$\vec{\mathbf{N}}' = \frac{K_T}{d^2}\vec{\mathbf{N}}^*,\ \theta = \beta d\,\theta^*,\ K_T = \frac{\chi_T}{\rho_0 C_v}$, Where K_T is the thermal diffusivity, equations (11) to (15) in non-dimensional form after dropping stars become

$$\frac{\partial\vec{\mathbf{q}}}{\partial t} = -\nabla p + R\theta\hat{\mathbf{e}}_z + (1+K)\nabla^2\vec{\mathbf{q}} + K\nabla\times\vec{\mathbf{N}} + Q(\nabla\times\vec{\mathbf{h}})\times\hat{\mathbf{e}}_x \tag{16}$$

$$\bar{\mathbf{j}}\,\frac{\partial\vec{\mathbf{N}}}{\partial t} = C'\nabla\,(\nabla.\vec{\mathbf{N}}) - C\,\nabla\times\,\nabla\times\,\vec{\mathbf{N}} + K(\nabla\times\vec{\mathbf{q}} - 2\vec{\mathbf{N}}) \tag{17}$$

$$P_r\frac{\partial\theta}{\partial t} = \nabla^2\theta + W - \bar{\boldsymbol{\delta}}\xi \tag{18}$$

$$P_r\frac{\partial\vec{\mathbf{h}}}{\partial t} = \frac{\partial\vec{\mathbf{q}}}{\partial x} + \frac{P_r}{P_m}\nabla^2 h \tag{19}$$

Where $R = \frac{\rho_0 g\alpha\beta d^4}{\mu K_T}$ is the thermal Rayleigh number, $Q = \frac{\mu_e H_0^2 d^2}{4\pi\mu K_T}$ is the Chandrasekhar number, $\xi = (\nabla\times\vec{\mathbf{N}})z,\ W = \vec{\mathbf{q}}.\hat{e}_z,\ P_r = \frac{\mu}{\rho_o K_T}$ is the Prandtl number, $P_m = \frac{\mu}{\rho_o\gamma_m}$ is the Magnetic Prandtl number and $C' = \frac{\alpha'+\beta'+\gamma'}{\mu d^2},\ C = \frac{\gamma'}{\mu d^2},\ K = \frac{\zeta}{\mu}$.

5. Boundary Conditions:

Both the boundaries of the layer are taken to be free and heat conducting so that the boundary conditions are as follows:

$$w = 0 = \frac{\partial^2 w}{\partial z^2},\ \vec{\mathbf{N}} = 0,\ \theta = 0 \text{ at } z = 0 \text{ and } z = 1,\ \zeta = 0 \text{ at } z = 0, 1 \tag{20}$$

6. Dispersion Equations:

Applying curl operator to equations (16) to (19), we have the following dispersion equations

$$\frac{\partial}{\partial t}\nabla^2 w = R\nabla_1^2\theta + (1+K)\nabla^4 w + K\nabla^2\xi + Q\frac{\partial}{\partial x}(\nabla^2 h_z) \quad (21)$$

$$\left\{\left[\bar{\mathbf{j}}\frac{\partial}{\partial t} - C\,\nabla^2 + 2K\right]\left[\frac{\partial}{\partial t} - (1+K)\nabla^2\right] + K^2\nabla^2\right\}\zeta_z = Q\left[\bar{\mathbf{j}}\frac{\partial}{\partial t} - C\nabla^2 + 2K\right]\frac{\partial m_z}{\partial x} \quad (22)$$

$$\bar{\mathbf{j}}\,\frac{\partial\xi}{\partial t} = C\,\nabla^2\xi - K(\nabla^2 w + 2\xi) \quad (23)$$

$$P_r\frac{\partial m_z}{\partial t} = \frac{\partial\zeta_z}{\partial x} + \frac{P_r}{P_m}\nabla^2 m_z \quad (24)$$

$$P_r\frac{\partial\theta}{\partial t} = \nabla^2\theta + w - \bar{\boldsymbol{\delta}}\,\xi \quad (25)$$

$$P_r\frac{\partial h_z}{\partial t} = \frac{\partial w}{\partial x} + \frac{P_r}{P_m}\nabla^2 h_z \quad (26)$$

Where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $h_z = \vec{\mathbf{h}}.\,\vec{\mathbf{e}}_z$, $\zeta_z = (\nabla\times\vec{\mathbf{q}}).\,\hat{e}_z$, $m_z = (\nabla\times\vec{\mathbf{h}}).\,\hat{e}_z$

7. Normal Mode Analysis:

In normal mode analysis we take

$$[w, \zeta_z, \xi, \theta, m_z, h_z] = [W(z), X(z), G(z), \Theta(z), M(z), B(z)]\; e^{i(k_x x + k_y y) + \sigma t} \quad (27)$$

Therefore, $\nabla \equiv D^2 - a^2$, $\nabla_1^2 = -a^2$, where $a = \sqrt{k_x^2 + k_y^2}$ is the wave number and σ is stability parameter.

Using equation (27), equations (21) to (26) become

$$\left[\sigma(D^2 - a^2) - (1+K)(D^2 - a^2)^2\right]W = -Ra^2\Theta + K(D^2 - a^2)G + ik_x\,Q(D^2 - a^2)B \quad (28)$$

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$$\left[\bar{j}a - C(D^2 - a^2) + 2K\right]G = -K(D^2 - a^2)W \tag{29}$$

$$\left\{\left[\bar{j}\sigma - C(D^2 - a^2) + 2K\right]\left[\sigma - (1+K)(D^2 - a^2)\right] + K^2(D^2 - a^2)\right\}X$$

$$= Q\,i\,k_x[\bar{j}\,\sigma - C(D^2 - a^2) + 2K]M \tag{30}$$

$$\frac{P_r}{P_m}\left[P_m\,\sigma - (D^2 - a^2)\right]M = ik_x\,X \tag{31}$$

$$\left[P_r\,\sigma - (D^2 - a^2)\right]\Theta = W - \bar{\boldsymbol{\delta}}G \tag{32}$$

$$\frac{P_r}{P_m}\left[P_m\ \sigma - (D^2 - a^2)\right]B = i\,k_x\,W \tag{33}$$

Boundary conditions (20) becomes

$$\left.\begin{array}{l} W = D^2W = \Theta = G = 0,\ M = 0 \\ DX = 0, DM = 0, B = 0 \\ \text{at } z = 0 \text{ and } z = 1 \end{array}\right] \tag{34}$$

Using (34), equations (28) to (33) yield

$$D^4W = 0 = D^2B = D^2G = D^2M = D^2\Theta \ \text{ at } \ z = 0 \ \text{ and } z = 1 \tag{35}$$

Using the boundary conditions (34) and (35), we obtain

$$D^{2n}W = 0 \ \text{ at } \ z = 0, 1$$

Where n is a positive integer.

Thus, the proper solution W characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{36}$$

Where W_0 is a constant.

Eliminating Θ, G, M, X, B between (28) to (33), and substitute for *W*, we obtain

$$\left[\sigma b+(1+K)b^2\right]\left[\bar{\mathbf{j}}\,\sigma+Cb+2K\right]\left[P_r\sigma+b\right]\frac{P_r}{P_m}\left[P_m\sigma+b\right]$$

$$=Ra^2\frac{P_r}{P_m}\left[P_m\,\sigma+b\right]\left[\,\bar{\mathbf{j}}\,\sigma+Cb+2K-\bar{\boldsymbol{\delta}}Kb\right]+K^2b^2\left[P_r\sigma+b\right]\frac{P_r}{P_m}\left[P_m\sigma+b\right]$$

$$-b\,k_x^2\,Q\,[\bar{\mathbf{j}}\sigma+Cb+2K]\,[P_r\sigma+b] \tag{37}$$

Where $b=\pi^2+a^2$

8. Marginal Instability:

For the Stationary Marginal State, we set $\sigma=0$ in (37), we obtain

$$R=\frac{b^3[C(1+K)b+2K+K^2]+\dfrac{bk_x^2P_m}{P_r}Q(Cb+2K)}{a^2(Cb+2K-\bar{\boldsymbol{\delta}}Kb)} \tag{38}$$

Equation (38) can be written as

$$R=\frac{P_rb^3[2A+b+K(A+b)]+b^2k_x^2P_mQ(2A+b)}{a^2P_r(2A+b-\bar{\boldsymbol{\delta}}Ab)} \tag{39}$$

Where $A=\dfrac{K}{C}$.

In order to investigate the effects of magnetic field *Q*, coupling parameter *K*, micropolar heat conduction parameter $\bar{\delta}$ and micropolar coefficient *A*, we examine the behaviour of $\dfrac{dR}{dQ}, \dfrac{dR}{dK}, \dfrac{dR}{d\bar{\delta}}$ and $\dfrac{dR}{dA}$.

From (39), we have

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$$\frac{dR}{dQ} = \frac{b^2 k_x^2 P_m (2A+b)}{a^2 P_r (2A+b-\overline{\delta} Ab)}$$

which is always positive if $\overline{\boldsymbol{\delta}} < \frac{1}{A}$ thus, the magnetic field has stabilizing effect if $\overline{\boldsymbol{\delta}} < \frac{1}{A}$

Again from (39), we have

$$\frac{dR}{dK} = \frac{b^3 (A+b)}{a^2 (2A+b-\overline{\boldsymbol{\delta}} Ab)}$$

which is always positive if $\overline{\boldsymbol{\delta}} < \frac{1}{A}$ thus, the coupling parameter K has a stabilizing effect, if $\overline{\delta} < \frac{1}{A}$.

Again from (39), we have

$$\frac{dR}{d\overline{\delta}} = \frac{Ab\left[b^2 k_x^2 P_m\, Q(2A+b) + P_r\, b^3 \{2A+b+K(A+b)\} \right]}{a^2 P_r\, (2A+b-\overline{\boldsymbol{\delta}} Ab)^2}$$

which is always positive

Thus, the micropolar heat conduction parameter $\overline{\delta}$ has stabilizing effect.

Again from (39), we have

$$\frac{dR}{dA} = \frac{b^5 \overline{\boldsymbol{\delta}}\, P_r (1+K) + b^4 \{-P_r K + \overline{\boldsymbol{\delta}} k_x^2 P_m Q\}}{a^2 P_r (2A+b-\overline{\delta} Ab)^2}$$

Now $\frac{dR}{dA} > 0$ if $Q > \frac{P_r\, K}{\overline{\boldsymbol{\delta}}\, k_x^2 P_m}$

Thus, the micropolar coefficient A has stabilizing effect if $Q > \frac{P_r K}{\overline{\boldsymbol{\delta}}\, k_x^2\, P_m}$.

In the absence of $\overline{\boldsymbol{\delta}}\, (\overline{\boldsymbol{\delta}} = 0)$, $\frac{dR}{dA} < 0$, which predicts that the micro coefficient A has destabilizing effect.

9. Conclusions (Stationary Convection):

i. The magnetic field has stabilizing effect when $\overline{\delta} < \frac{1}{A}$.

ii. The coupling parameters K has a stabilizing effect if $\overline{\delta} < \frac{1}{A}$.

iii. The micropolar heat conduction parameter $\bar{\delta}$ has stabilizing effect.

iv. The micropolar coefficient *A* has stabilizing effect when $Q > \frac{P_r K}{\bar{\delta} k_x^2 P_m}$.

v. In the absence of micropolar heat conduction parameter, the micropolar coefficient has destabilizing effect.

10. Graphical Representation:

The variation of thermal Rayleigh number R with respect to Magnetic field (Q), Coupling parameter (K), Micropolar heat conduction parameter ($\bar{\delta}$) and Micropolar coefficient (A) respectively are shown by the following diagrams:

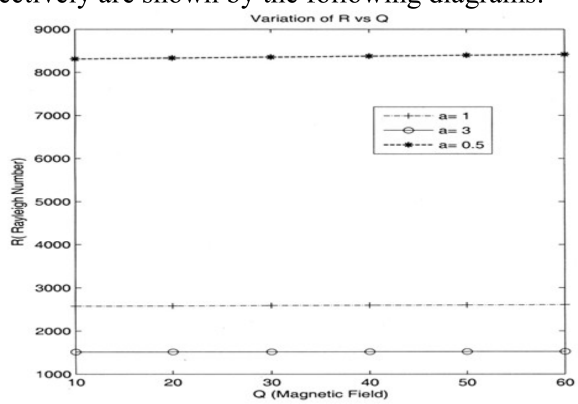

Fig. 2 Marginal instability curve for the variation of R vs Q for A=0.5, K=1, P_r=2, P_m=4, $\bar{\delta}$ = 0.5, k_x=0.5

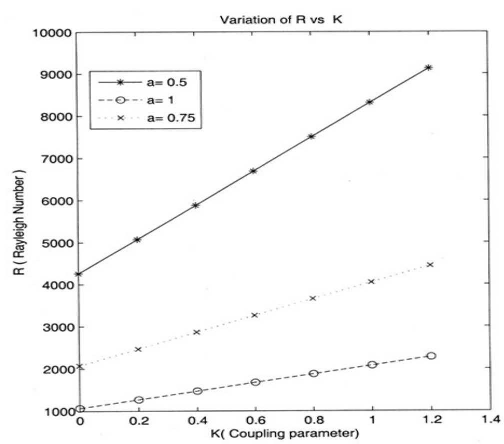

Fig.3 Marginal instability curve for the variation of R vs K for A=0.5, P_r=2, P_m=4, Q=10, k_x=0.05, = 0.5.

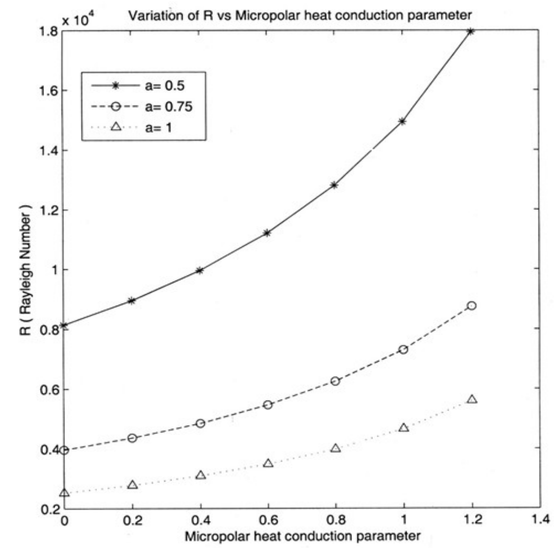

Fig. 4 Marginal instability curve for the variation of R vs for A=0.5, P_r=2, P_m=4, K=1, k_x=0.05, Q=20.

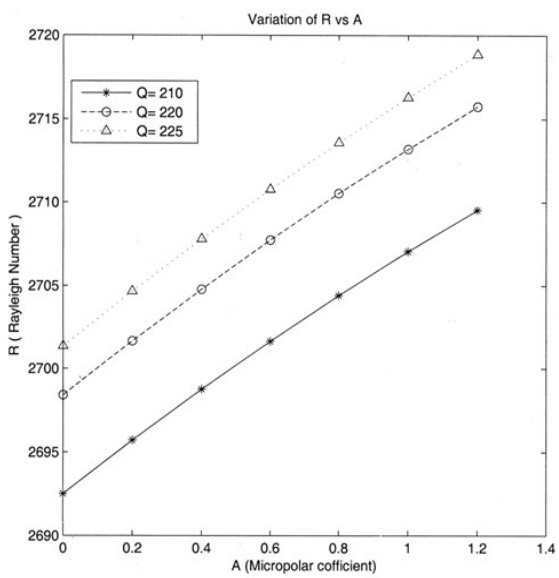

Fig. 5 Marginal instability curve for the variation of R vs A for a=1, P_r=2, P_m=4, K=1, k_x=0.05, =0.05.

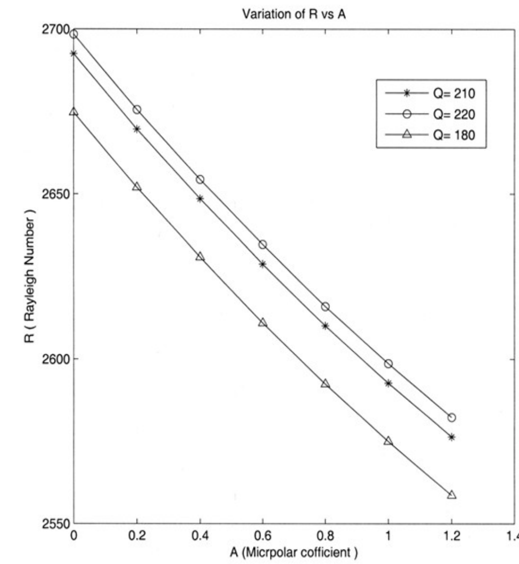

Fig. 6 Marginal instability curve for the variation of R vs A for =0, P_r=2, P_m=4, K=1, k_x=0.05, a=1.

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